2D LOCAL EXTERIOR SOUND FIELD REPRODUCTION USING AN ADDITION THEOREM BASED ON CIRCULAR HARMONIC EXPANSION

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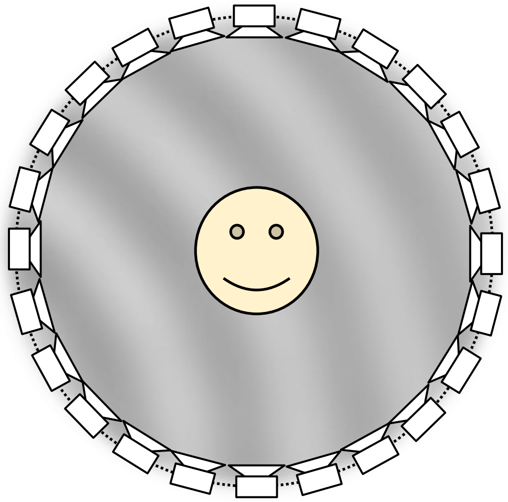
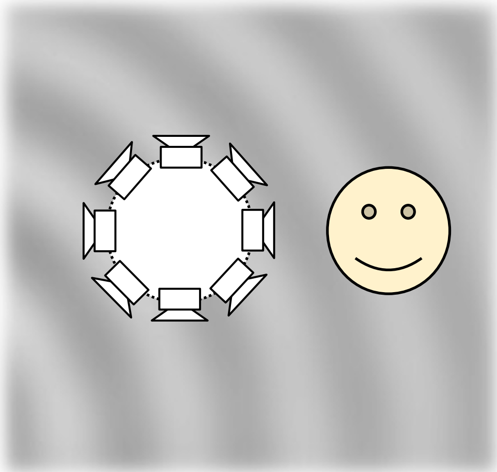
BACKGROUND

Sound field reproduction

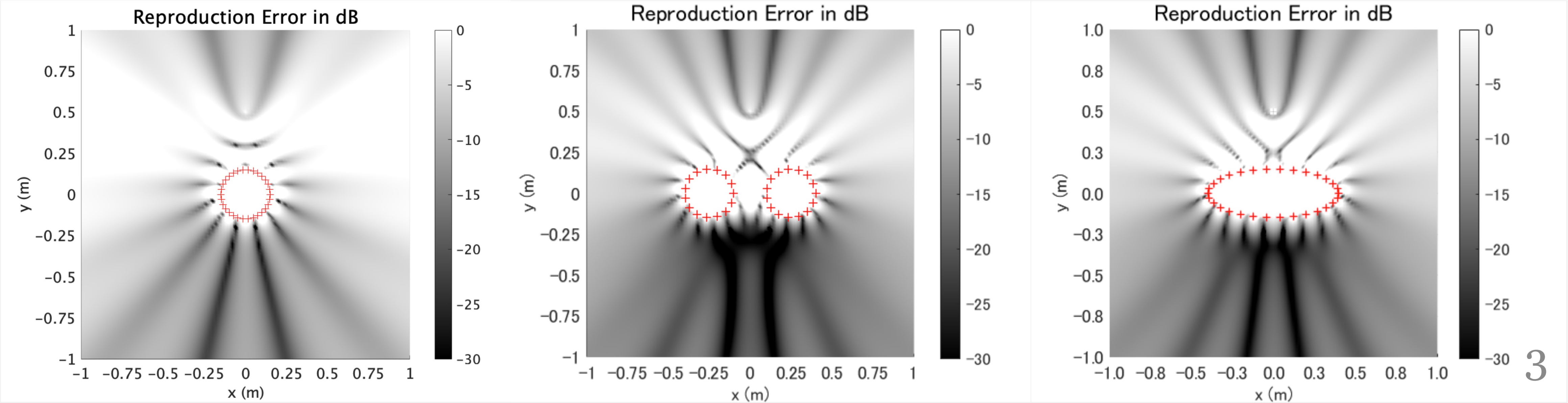
- 3D audio, immersive audio
- A closed boundary divides a space into two (Kirchhoff-Helmholtz integral equation)
- Either interior or exterior sound field can be reproduced





EXTERIOR SOUND FIELD REPROUDCTION

- Examples: Focused Source Reproduction (FSR), radiation control
- Compact loudspeaker arrays
  - ➢Difficult to achieve a high performance
- We proposed array geometries for FSR (Ren+, 2018; 2020).
  - Expecting further improvements



CONCEPT

Controlling the whole exterior area was difficult.

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Performance can be improved with smaller areas?

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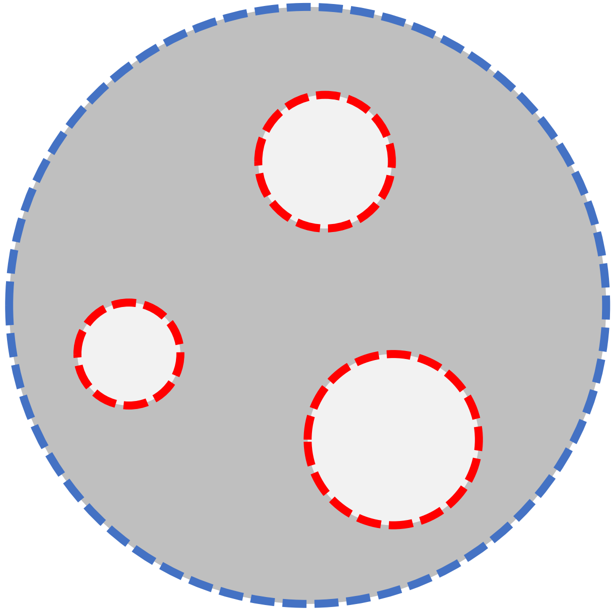
Try focusing on several local areas of the exterior sound field.

MULTIZONE REPRODUCTION

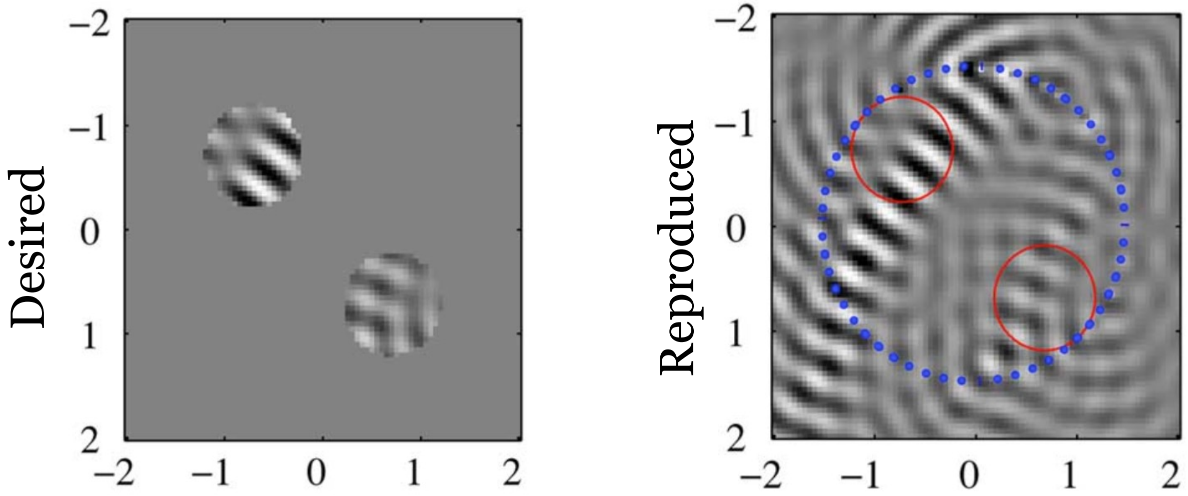
For interior sound field reproduction

- Conventional: whole area inside blue boundary
- Multizone: local areas inside red boundary

- Methods: pressure-matching; mode-matching (based on additional theorem)



(Wu+, 2010)

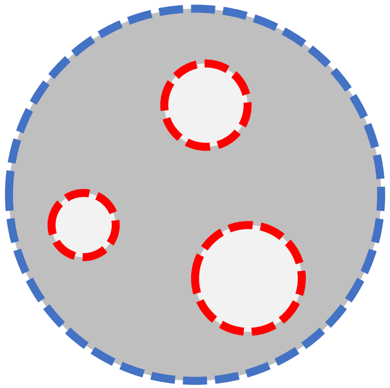


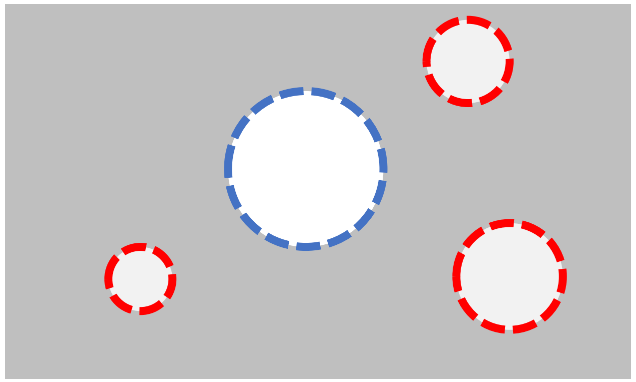
Improves performance at local areas

CONTRIBUTIONS OF THE STUDY

- A multizone reproduction methods for exterior sound field.
- Wave domain process based on circular harmonic expansion.
  - Conv: interior field→ interior field (Bessel → Bessel)
  - Prop: exterior field→ interior field (Hankel→ Bessel)

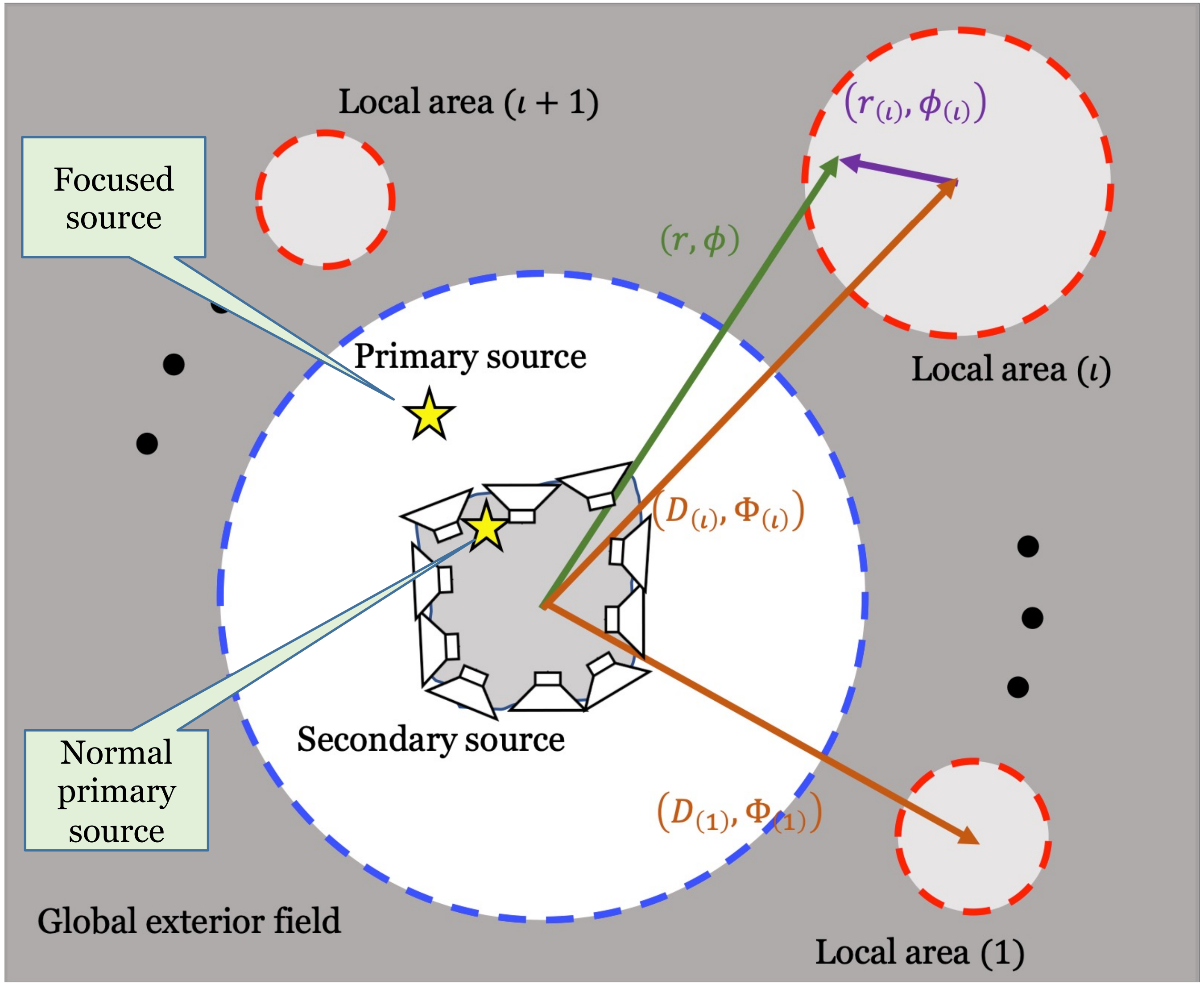
Bessel: arriving wave
Hankel: radiating wave





- Discussions on limitations.
- Numerical simulations and experiments on FSR.

MATHEMATICAL MODEL



- Primary and secondary sources inside blue boundary
  - ➢Sound field outside the boundary can be reproduced.
- Conventional: control region is the global exterior field.
- Proposal: We set local areas (red circles); control regions are the interior field of them.

CONVENTIONAL METHOD (CIRCULAR HARMONIC EXPANSION BASED)

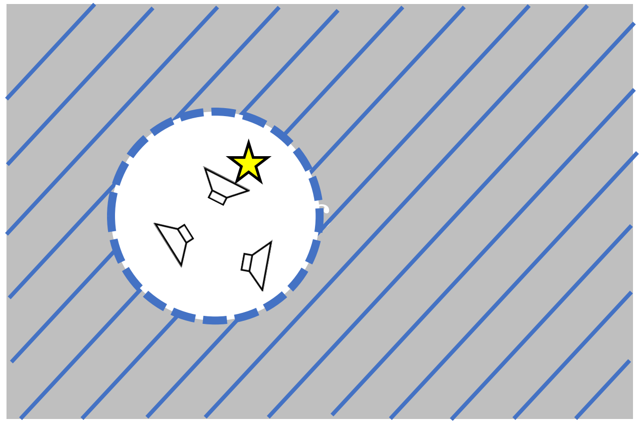
- The primary sound field (target field)

$$p(\boldsymbol{r},\omega) = \sum_{\nu=-\infty}^{\infty} \alpha_\nu H_\nu^{(2)}(kr)e^{j\nu\phi},$$

- The secondary sound field (reproduced field)

$$\hat{p}(\boldsymbol{r},\omega) = \sum_{l=1}^{L} G(\boldsymbol{r}|\boldsymbol{r}_l,\omega)d_l$$

$$\hat{p}(\boldsymbol{r},\omega) = \sum_{\mu=-\infty}^{\infty} \xi_\mu(\boldsymbol{d})H_\mu^{(2)}(kr)e^{j\mu\phi},$$



- Using the orthogonality of the expansion basis

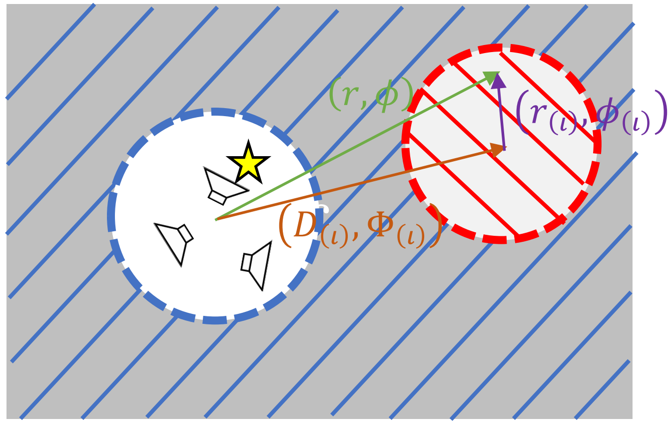
$$\xi_\nu(\boldsymbol{d}) = \alpha_\nu$$

*For a circular loudspeaker array, $\boldsymbol{d}$ can be derived easily with $\xi_\nu(\boldsymbol{d})$.

LOCAL SOUND FIELD REPRODUCTION

- The primary sound field of (the ι-th) local area

$$p(\boldsymbol{r}_{(\iota)}) = \sum_{\nu=-\infty}^{\infty} \alpha_{\nu,(\iota)} J_\nu(kr_{(\iota)}) e^{j\nu\phi_{(\iota)}},$$



- The secondary field was a series of Hankel function

$$\hat{p}(\boldsymbol{r},\omega) = \sum_{\mu=-\infty}^{\infty} \xi_\mu(\boldsymbol{d}) H_\mu^{(2)}(kr) e^{j\mu\phi}$$

  - A Hankel-Bessel transformation required.

- Graf's addition theorem:

$$H_\mu^{(2)}(kr)e^{j\mu\phi} = \sum_{\nu=-\infty}^{\infty} \underbrace{H_{\mu-\nu}^{(2)}(kD_{(\iota)})e^{j(\mu-\nu)\Phi_{(\iota)}}}_{T_{\nu,\mu,(\iota)}} J_\nu(kr_{(\iota)})e^{j\nu\phi_{(\iota)}}$$

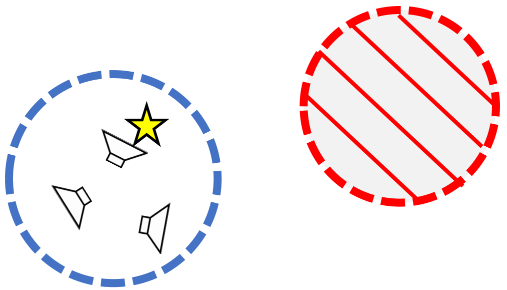
LOCAL SOUND FIELD REPRODUCTION

- The secondary sound field of the local area

$$\hat{p}(\boldsymbol{r}) = \sum_{\nu=-\infty}^{\infty}\sum_{\mu=-\infty}^{\infty} T_{\nu,\mu,(\iota)}\xi_{\mu}(\boldsymbol{d}) J_{\nu}\left(kr_{(\iota)}\right) e^{j\nu\phi_{(\iota)}}$$

- Using the orthogonality, we derive

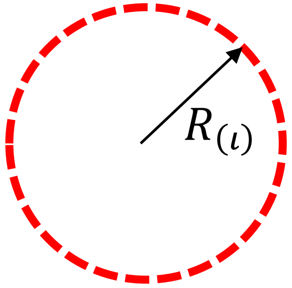
$$\alpha_{\nu,(\iota)} = \sum_{\mu=-\infty}^{\infty} T_{\nu,\mu,(\iota)}\xi_{\mu}(\boldsymbol{d})$$



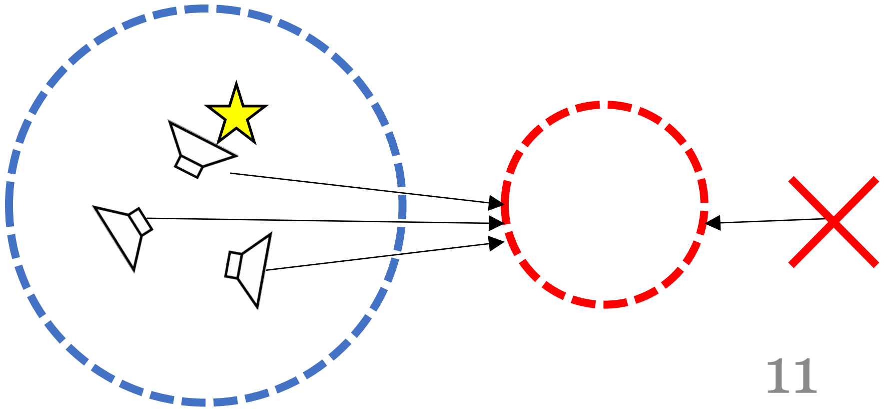
Finally, to control all local areas at the same time, the solution can be obtained by a least-squares method.

LIMITATIONS

- Area size
  - Determined by truncation order
  - Maximum radius $R_{(\iota)}$: $N_{(\iota)} \geq \left\lceil \frac{ekR_{(\iota)}}{2} \right\rceil$ (Kennedy+, 2007)
  - Number of loudspeakers L: $\sum_{\iota} N_{(\iota)} \leq M \leq \left\lfloor \frac{L-1}{2} \right\rfloor$ (well-conditioned)

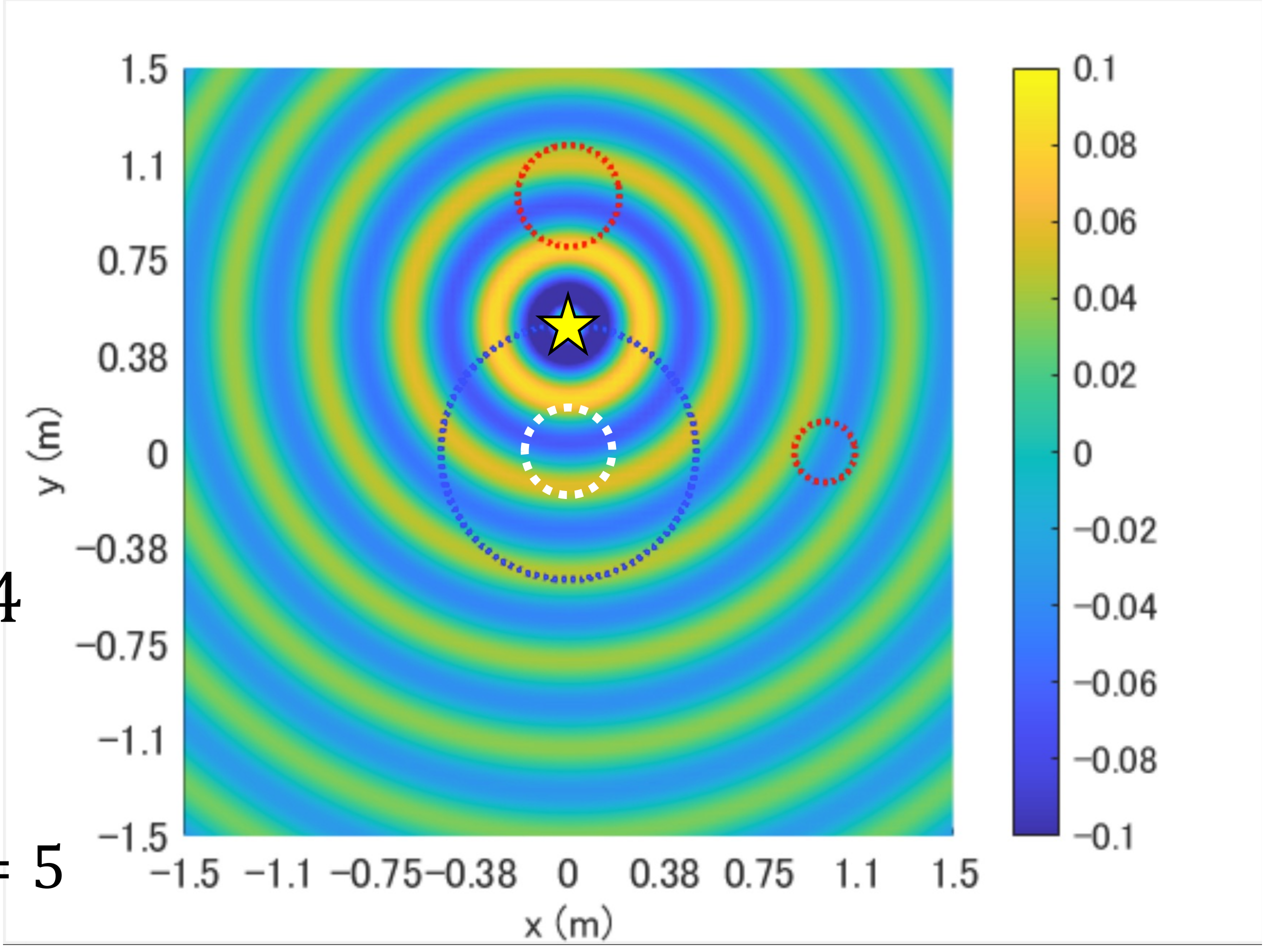


- Irreproducible sound fields
  - Both primary and secondary sources are outside the local area.
  - The local area is not fully-surrounded by the secondary sources.
    - ➢The target field the opposite direction of the secondary sources is difficult to be reproduced.



NUMERICAL SIMULATION: LOCAL SOUND FIELD REPRODUCTION

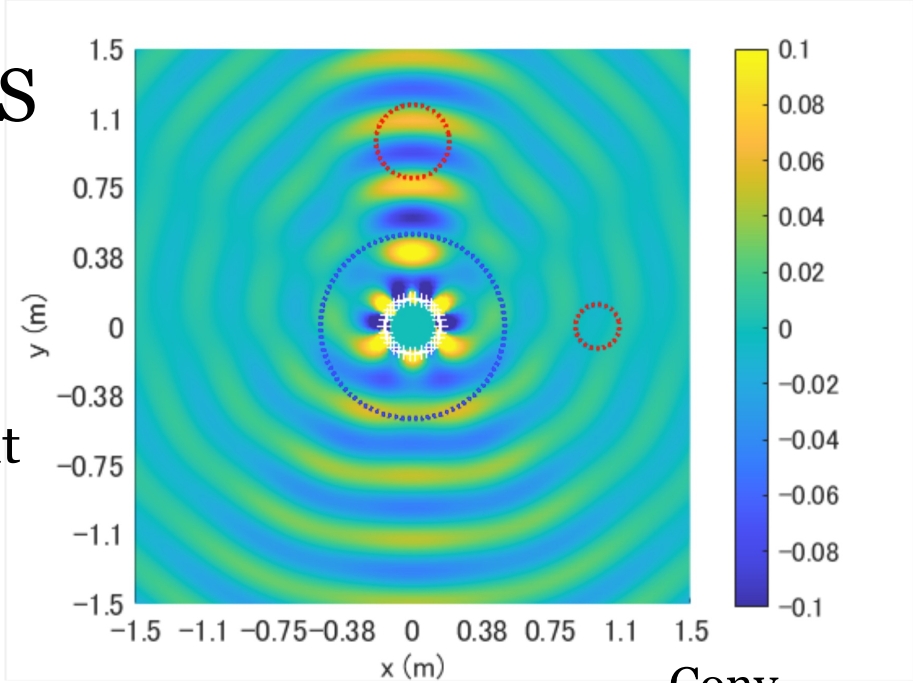
- Target source (focused source)
  - 1000 Hz omnidirectional source
  - Location: $(0.5\ \mathrm{m}, \frac{\pi}{2})$
- Secondary source (white circle)
  - Rigid circular loudspeaker array
  - Radius: 0.15 m; 30 loudspeakers
- Truncation order (total): $M = 14$
- Two local areas
  - Centers: $(1\ \mathrm{m}, 0)$, $(1\ \mathrm{m}, \pi/2)$
  - Truncation orders: $N_{(1)} = 3$, $N_{(2)} = 5$
  - ➢Area radii: 12 cm, 20 cm



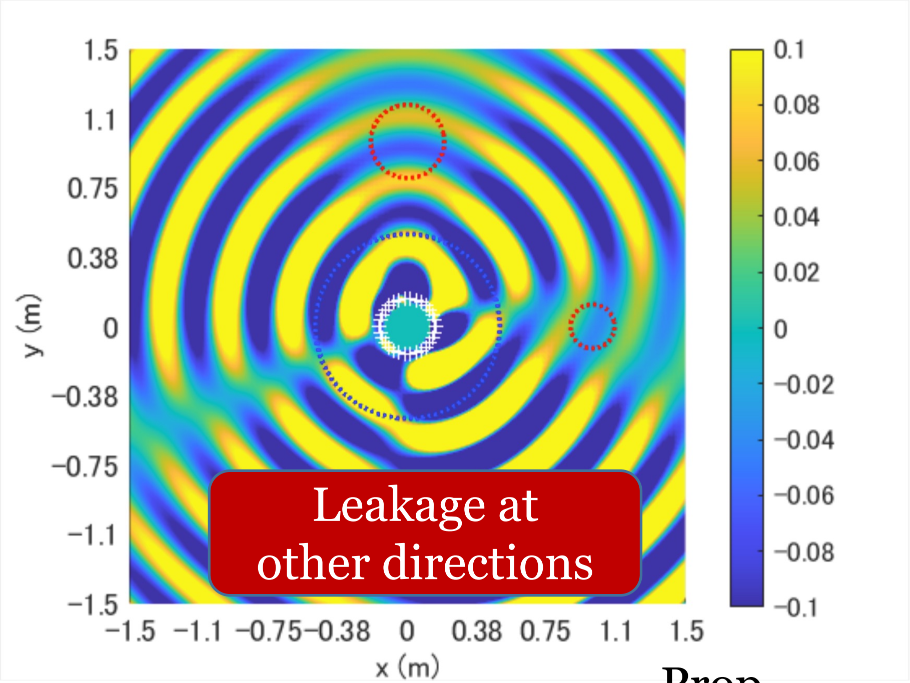
Control region: Conv outside blue circle
Prop inside red circle

RESULTS

wave front

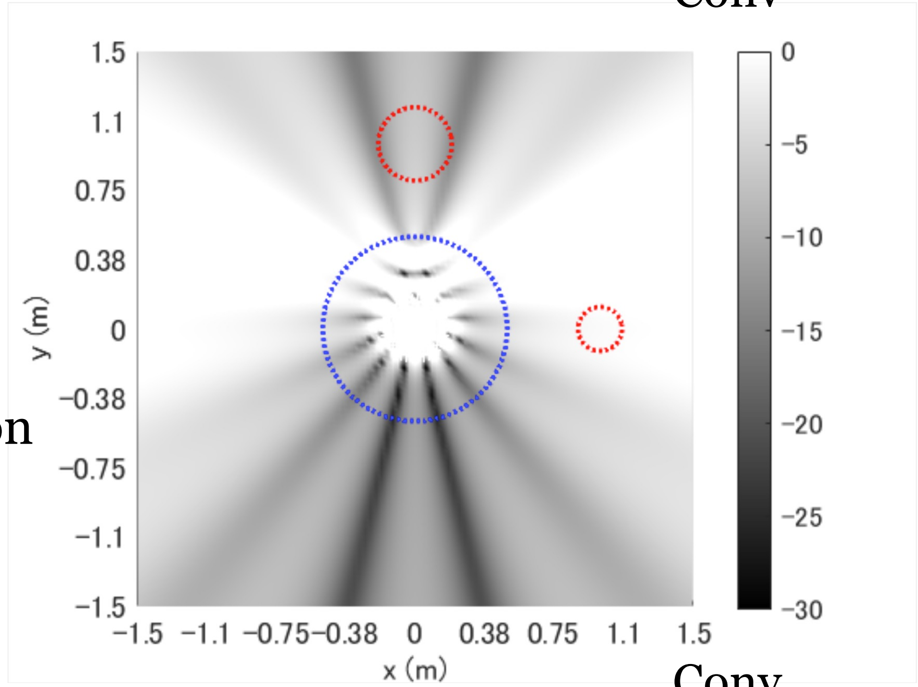


Conv

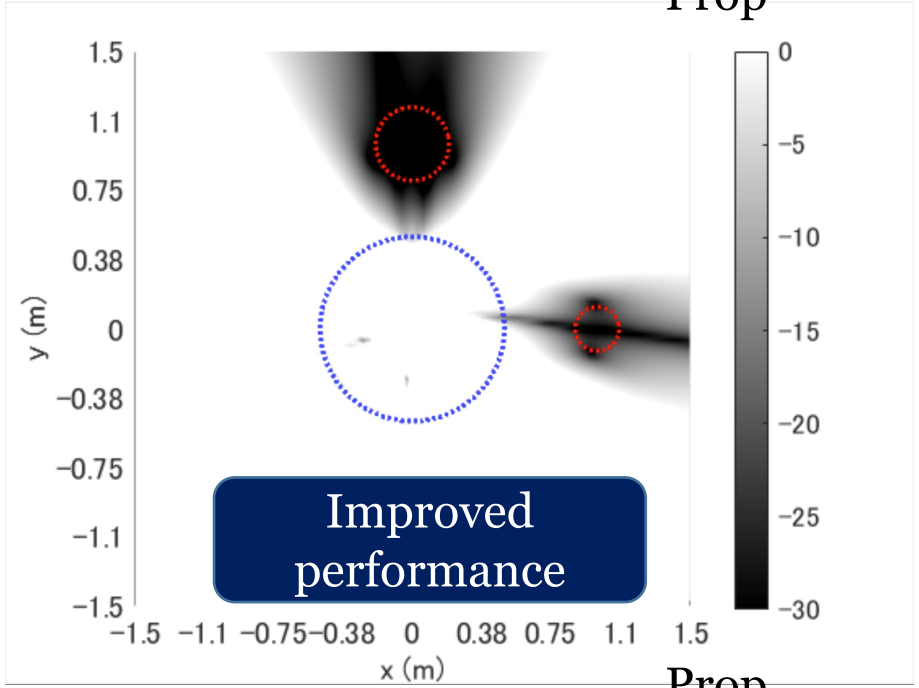


Prop

Reproduction error



Conv



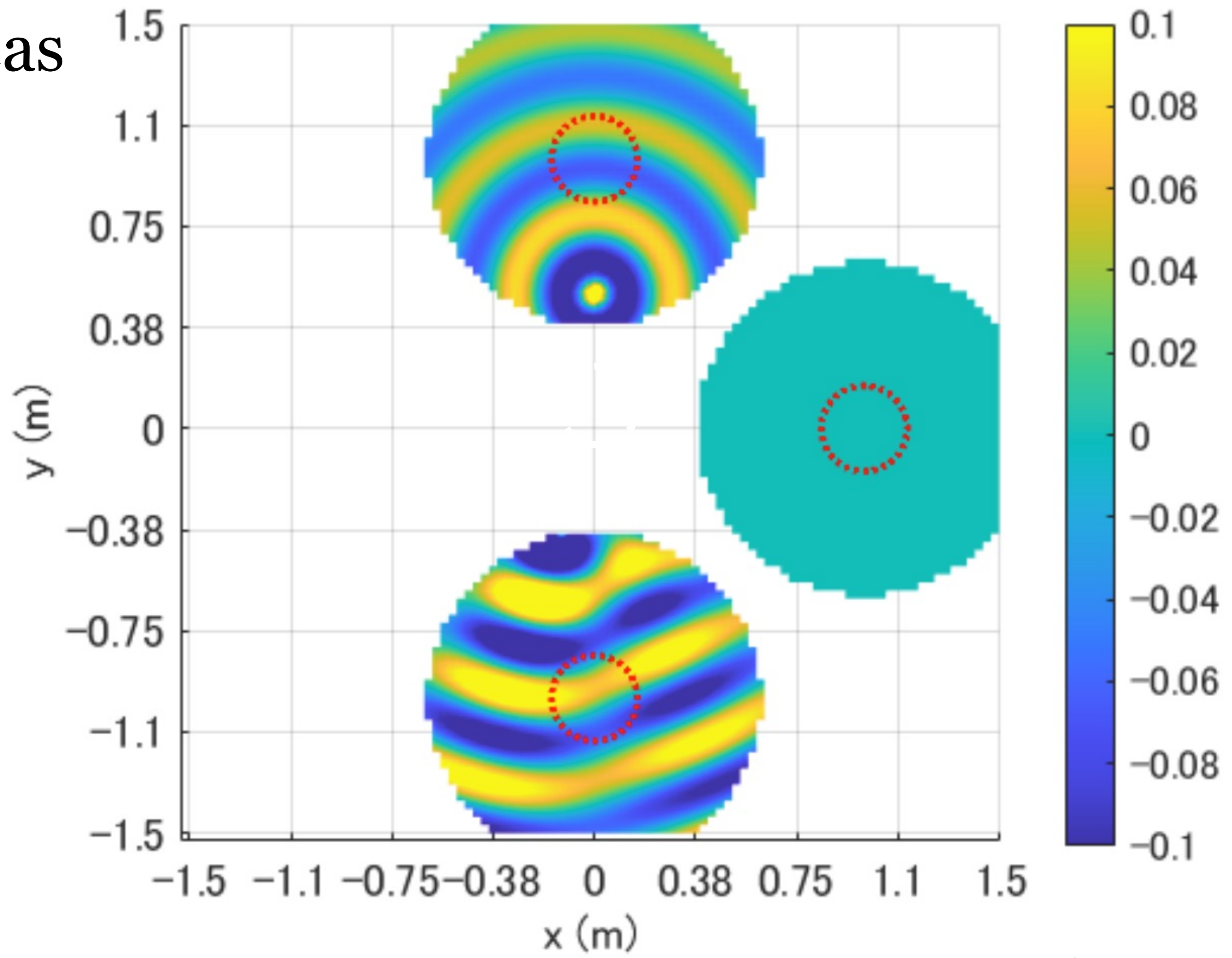
Prop

NUMERICAL SIMULATION: MULTIZONE REPRODUCTION

- Three sound fields / local areas
  1. $\left(1\ \mathrm{m}, \frac{\pi}{2}\right)$
     - Focused source
  2. $(1\ \mathrm{m}, 0)$
     - Dark / Quiet zone
  3. $(1\ \mathrm{m}, -\frac{\pi}{2})$
     - Sound field of 6 random sources

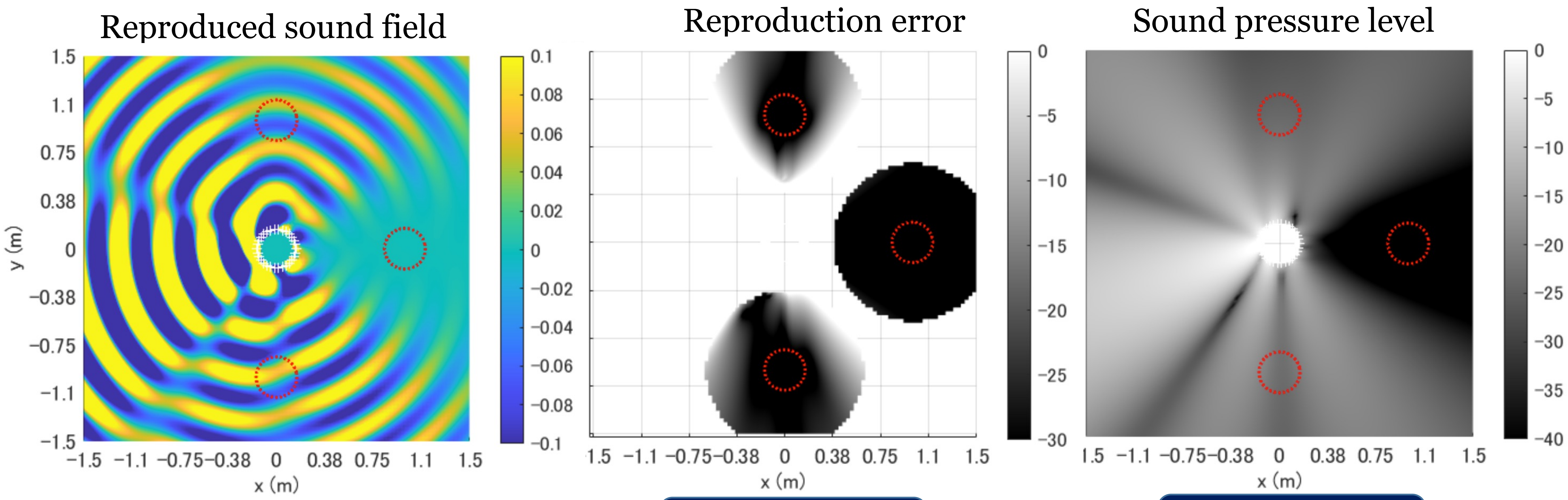
- Truncation orders: $N_{(\iota)} = 4$

➢Area radii: 16 cm



NUMERICAL SIMULATION: MULTIZONE REPRODUCTION

RESULTS



High accuracy

High contrast

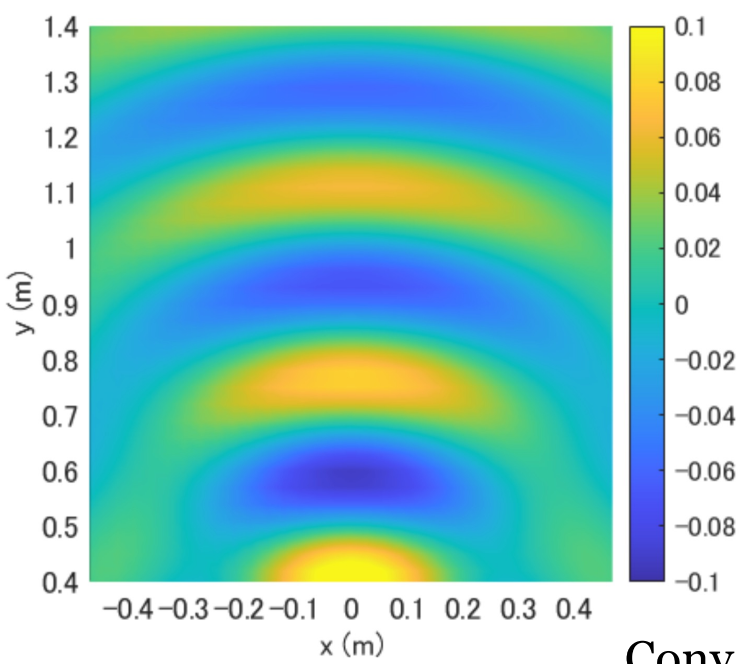
EXPERIMENTS

- Local sound field reproduction
  - Same target source as the simulation
- Secondary source
  - Rigid circular loudspeaker array
  - Radius: 0.15 m; 15 loudspeakers
  - Height: 0.572 m
- One local area at $(1\ \mathrm{m}, \pi/2)$

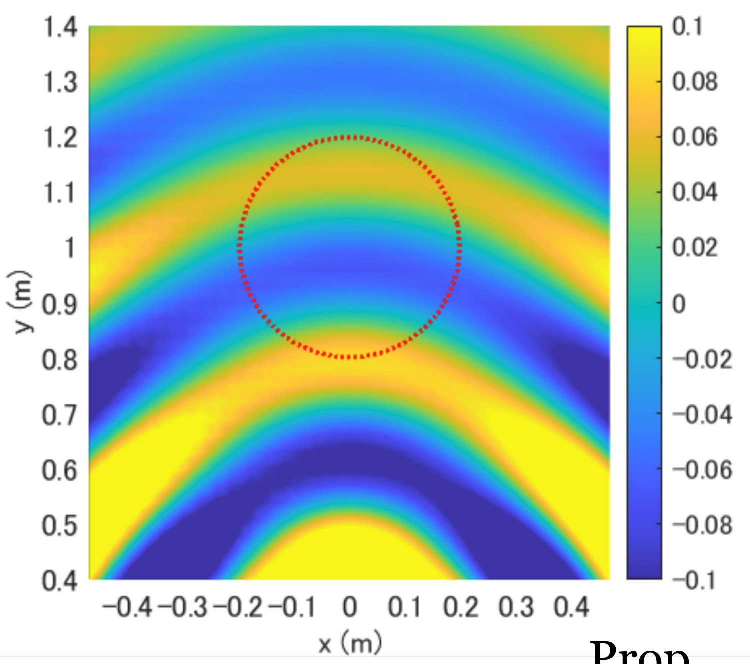
- Sound field measurement
  - Horizontal $0.94 \times 0.1\ \mathrm{m}^2$ sound field
  - Coordinates: $x \in [-0.47\ \mathrm{m}, 0.47\ \mathrm{m}]$, $y \in [0.4\ \mathrm{m}, 1.4\ \mathrm{m}]$
  - Impulse responses measured in 0.02 m grids

RESULTS

Theoretical

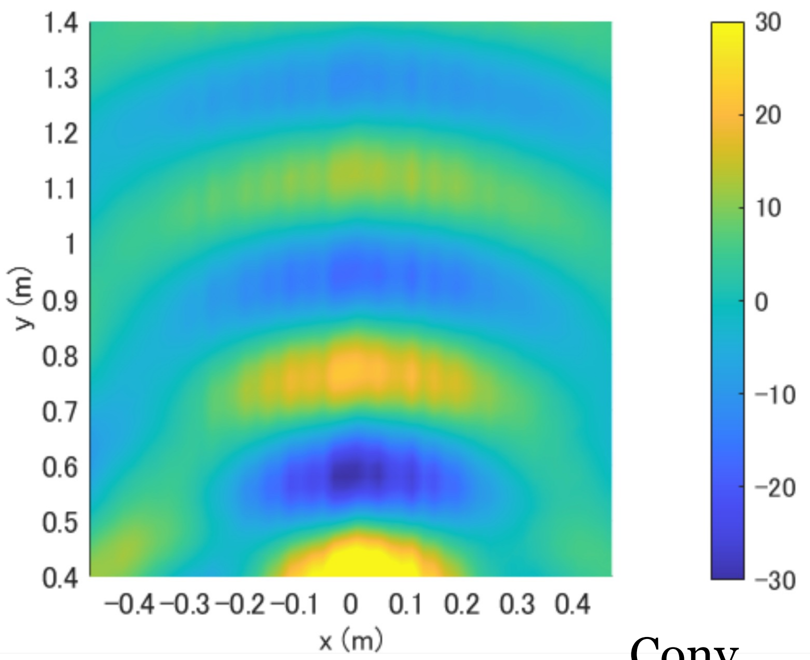


Conv

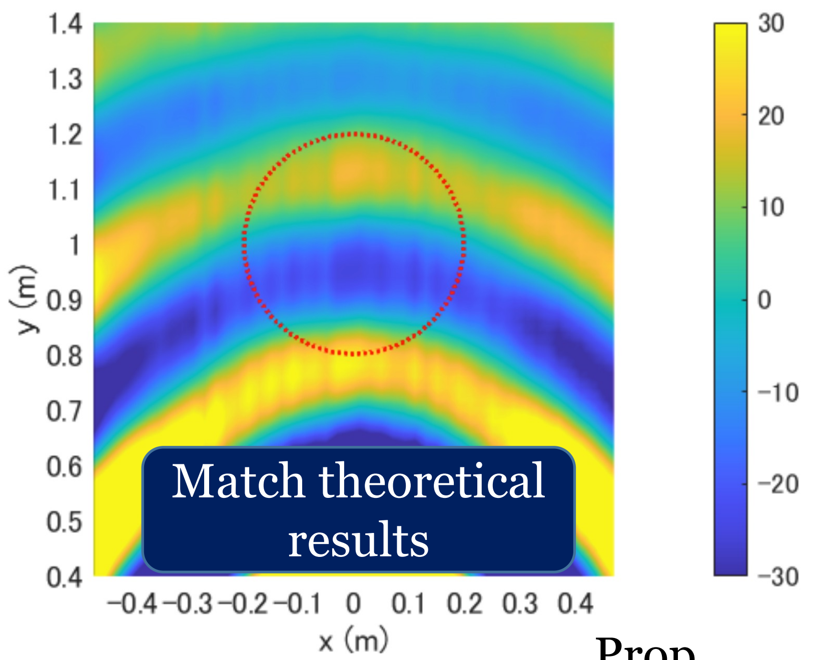


Prop

Experimental



Conv



Prop

SUMMARY

- In this study, we proposed a two-dimensional local exterior sound field reproduction method based on circular harmonic expansion.
- By applying the additional theorem, local interior areas of the global exterior field can be controlled.
- Simulation and experiment results show the method is valid for local sound field reproduction and multizone reproduction.
- 2.5D methods and reverberation-considered methods are considered as future works.

THANKS FOR LISTENING