2D sound field reproduction with elliptical loudspeaker array based on circular microphone array signals

Yi Ren[1], Kenta Imaizumi[2], Kimitaka Tsutsumi[2], and Yoichi Haneda[1]

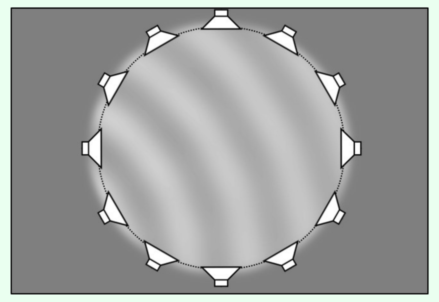
[1] The University of Electro-Communications [2] NTT Service Evolution Laboratories E-mail: ren.yi@uec.ac.jp Homepage: http://www.hanedalab.inf.uec.ac.jp/student/member/nin/index.html

Introduction

Sound Field Reproduction

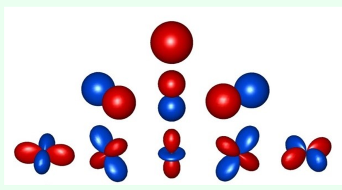
often implemented with loudspeaker arrays
(linear, circular, etc,.)

Circular Loudspeaker Array (CLA)



2D Higher Order Ambisonics (2D-HOA) based on

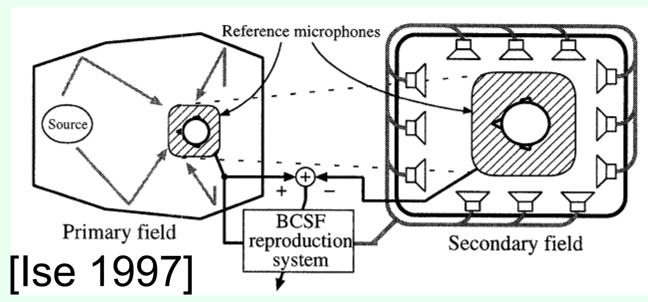
Circular Harmonics Expansion (CHE)



Pros: analytical driving function

Cons: does not fit rectangular rooms well

Artificial-shaped array



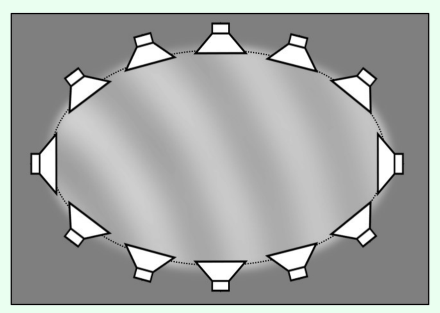
[Ise 1997]

Boundary Surface Control (BoSC)

Pros: can use an optimal shape

Cons: numerical solution (LMS)

Elliptical Loudspeaker Array (ELA)

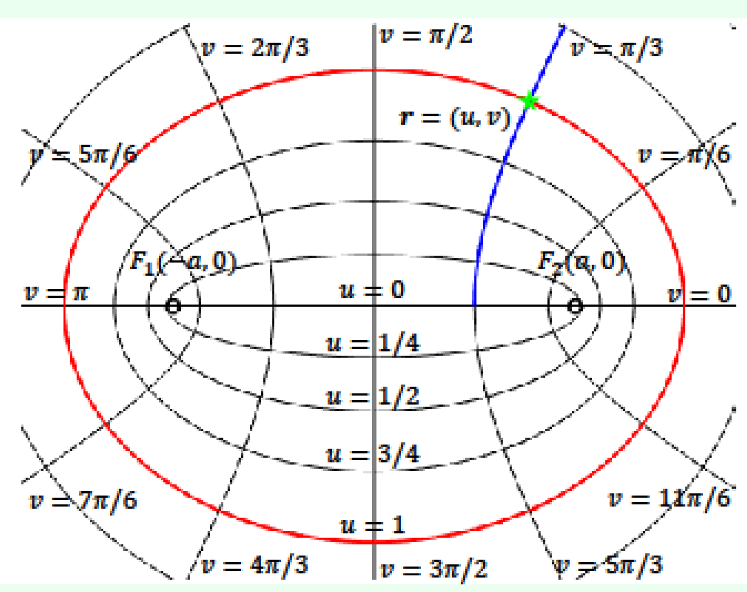


Fits normal room better than CLA.
Can be solved in an orthogonal coordinate system
Elliptical Harmonics Expansion (EHE) based on **Mathieu functions**
Analytical method: **mode-matching method**

Problem
Elliptical microphone array (EMA) recording
→
Proposal
Transfer circular microphone array (CMA) signals

Mathieu Functions

Elliptical coordinate system



Elliptical coordinate system

$$\begin{cases} x = a \cosh u \cos v \\ y = a \sinh u \sin v \end{cases}$$

$v \in [0, 2\pi]$, $u \in [0, \infty)$
$F_1(-a, 0), F_2(a, 0)$: Foci

Functions of an ellipse

$u = u_0$ (u_0: const)

Helmholtz equation

$$\nabla^2 \psi(v, u) + k^2 \psi(v, u) = 0$$

k: wave number

Laplace operator in elliptical coordinate system

$$\nabla^2 = \frac{1}{a^2(\cosh 2u - \cos 2v)} \left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} \right)$$

Separation of variables: $\psi(v, u) = N(v) \cdot M(u)$

Mathieu Function: a solution of Helmholtz equation

$N(v) = \{me_n(q, v)\}$: Mathieu angular function

$M(u) = \left\{ M_n^{(\zeta)}(q, u) | \zeta \in \{1,2,3,4\} \right\}$: Mathieu radial function

Sound field reproduction using ELA

Mode-matching method based on Mathieu functions

A sound field can be expanded by Mathieu function (EHE):

$$p(v, u) = \sum_{n=-\infty}^{\infty} \left[\alpha_n^{\mathrm{i}} M_n^{(1)}(q, u) me_n(q, v) + \alpha_n^{\mathrm{s}} M_n^{(4)}(q, u) me_n(q, v) \right]$$

For a sound field with no source inside the ELA, there are only incident field:

$$p(v, u) = \sum_{n=-\infty}^{\infty} \alpha_n M_n^{(1)}(q, u) me_n(q, v)$$

The secondary field reproduced by L loudspeakers on the curve $u = u_0$ is

$$\hat{p}(v, u) = \sum_{l=1}^{L} G(v, u | v_l, u_0) D(v_l)$$

(v_l, u_0): coordinates of lth loudspeaker; D: driving function; G: Green function for free field

Apply EHE to driving function and use the orthogonality of Mathieu function

$$\hat{p}(v, u) = \sum_{n=-\infty}^{\infty} \beta_n d_n M_n^{(1)}(q, u) me_n(q, v)$$

$$\beta_n = -\frac{j}{4} L M_n^{(4)}(q, u_0)$$

Match the coefficient of the original field and the secondary field by modes, driving function can be derived by

$$D(v_l) = \sum_{n=-\infty}^{\infty} d_n me_n(q, v_l) = \sum_{n=-\infty}^{\infty} \frac{\alpha_n}{\beta_n} me_n(q, v_l) \qquad (\mathbf{1})$$

Proposed Method

Conventional method for CLA (2D-HOA)

CMA signal → (CHE) → CHE coefficient → (Mode-matching) → Coefficient of driving function → (Inverse spatial Fourier transfer) → Driving function

Coefficient transform using additional theorem of Mathieu function

Method for ELA

EMA signal → (EHE) → EHE coefficient → (Mode-mathing) → Coefficient of driving function → (Inverse spatial Fourier transfer) → Driving function

Sound field recording using CMA (conventional method)

In studies of 2D-HOA[Poletti 2000], it is common to record fields with CMA.
CMA signals can be expanded by Bessel function (CHE):

$$p(r_0, \phi_i) = \sum_{m=-\infty}^{\infty} \eta_m J_m(kr_0) e^{jm\phi_i} \qquad (\mathbf{2})$$

(r_0, ϕ_i): coordinates of ith microphone; $J_m(z)$: Bessel function

Furthermore for a CMA with rigid baffle (to avoid the forbidden frequency):

$$\eta_m = \frac{j\pi k r_0 H_m^{(2)\prime}(kr_0)}{2\mathcal{M}} \sum_{i=1}^{\mathcal{M}} p(r_0, \phi_i) e^{-jm\phi_i}$$

$\mathcal{M}$: number of microphones; $H_m^{(2)\prime}(z)$: differential of Hankel function of the 2nd kind

Coefficient transform using additional theorem

With the mode-matching method for ELA, we can derive the driving function with $(\mathbf{1})$ if only the coefficient α_n is known.
Besides, a coefficient η_m can be derived from CMA signal.
It will become possible to apply CMA signal if we can transfer $\alpha_n \leftrightarrow \eta_m$

Additional theorem of Mathieu function

In 2-dimensional space, when polar coordinates (r, ϕ) and elliptical coordinates (u, v) show the same point, and $r > a$, we have

$$J_m(kr) e^{jm\phi} = \sum_{n=-\infty}^{\infty} \tau_{n,m}^{*} M_{m+n}^{(1)}(q, u) me_{m+n}(q, v)$$

$\tau_{n,m}^{*} = j^{-n} c_{-n}^{m+n}$ for even n and $\tau_{n,m}^{*} = 0$ for odd n; c_{-n}^{m+n}: coefficient used for calculating Mathieu functions

Apply additional theorem to $(\mathbf{2})$:

$$p(r_0, \phi_i) = \sum_{m=-\infty}^{\infty} \eta_m \sum_{n=-\infty}^{\infty} \tau_{n,m}^{*} M_{m+n}^{(1)}(q, u) me_{m+n}(q, v)$$

$$= \sum_{n'=-\infty}^{\infty} \left[\sum_{m=-\infty}^{\infty} \eta_m \tau_{n-m,m}^{*} \right] M_{n'}^{(1)}(q, u) me_{n'}(q, v)$$

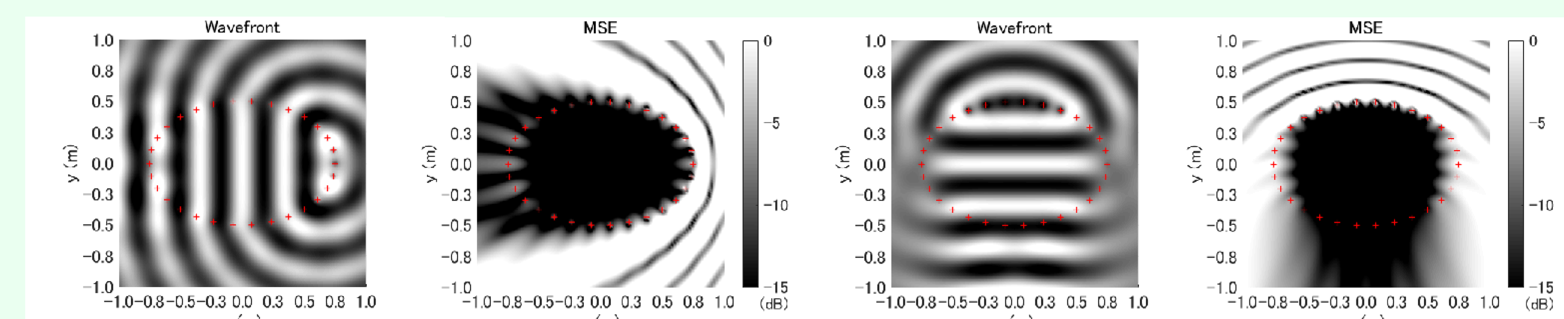
Coefficient $\tilde{\alpha}_{n'}$

Finally, substitute $\tilde{\alpha}_{n'}$ as α_n into $(\mathbf{1})$ to obtain the driving function

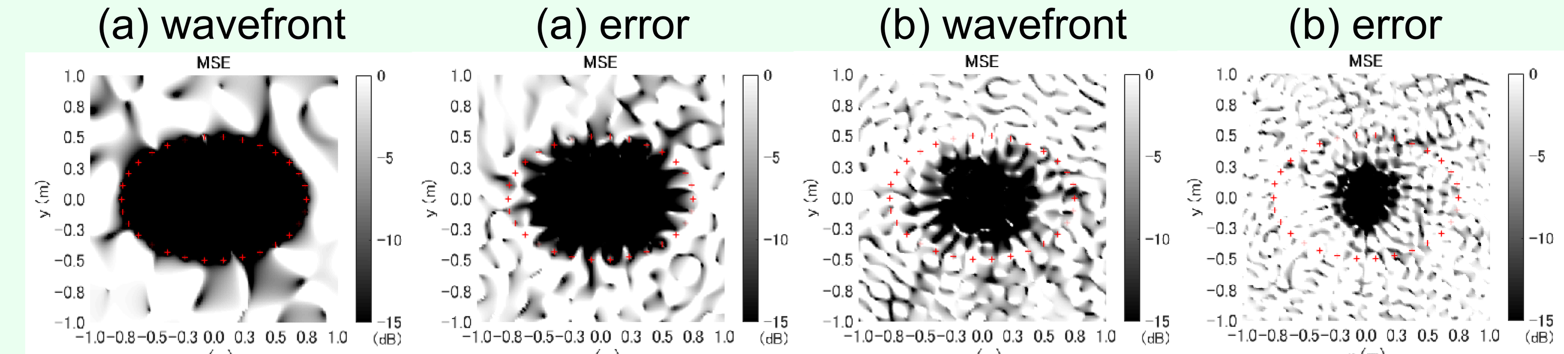
Numerical Simulation

Simulating conditions

$a = \sqrt{5}/4$ in the elliptical coordinate system.
ELA: 30 loudspeakers; 1.5 m major axis, 1.0 m minor axis; $u_0 \approx 0.80$; truncation order for EHE: 30.
CMA: 24 microphones: 0.3 m radius, truncation order for CHE: 11.
(a)1 kHz plane wave, arriving from $\varphi = 0$;
(b) 1 kHz plane wave, arriving from $\varphi = \pi/2$;
(c) 30 random cylindrical waves at c1:0.5 kHz, c2:1 kHz, c3:1.5 kHz, c4:2 kHz



(a) wavefront (a) error (b) wavefront (b) error



(c1) error (c2) error (c3) error (c4) error

Red crosses: loudspeaker location; normalized reproduction error: $\varepsilon = 10 \log_{10} \left(|\hat{P} - P|^2 / |P|^2 \right)$

Proposed method can reproduce interior sound field.
Listening area become smaller in higher frequencies

Summary

we proposed a method for sound field reproduction using ELA and CMA signals. In numerical simulations, the proposed method was valid for sound field reproduction. Studies about the listening area and truncation orders will be discussed in further studies.