How the distance and radius of two circular loudspeaker arrays affect sound field reproduction and directivity controls

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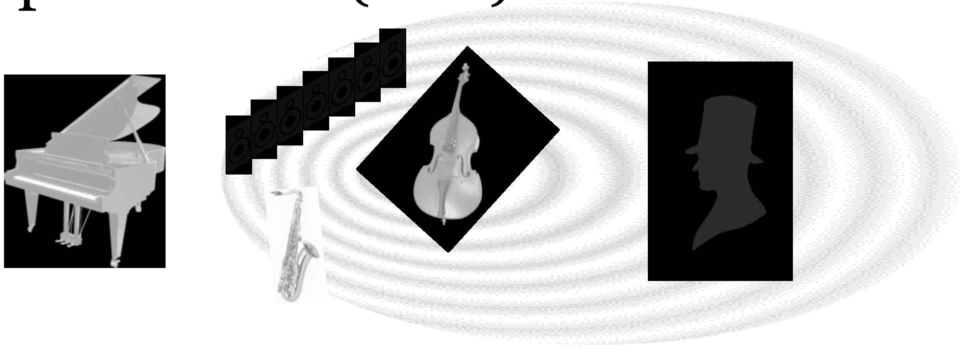
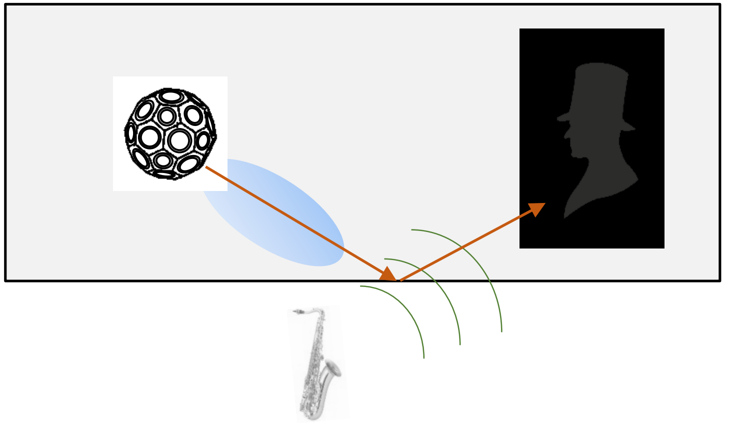
Introduction

- Sound field reproduction (SFR)



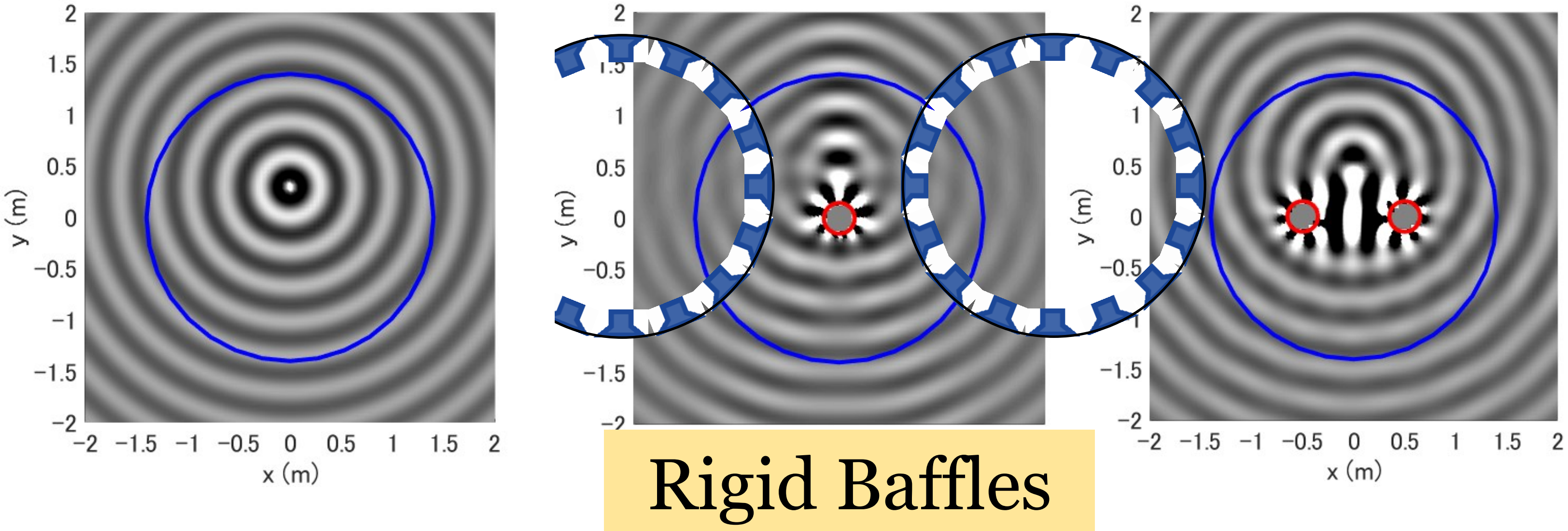
- Directivity control (DC)



- 2-D Sound Field
  - Circle -> Infinite Cylinder

Previous Works

- SFR using a Two Circular Loudspeaker Array (2CLA) model w/ comparison with a single Circular Loudspeaker Array (CLA) (Ren & Haneda, 2018)



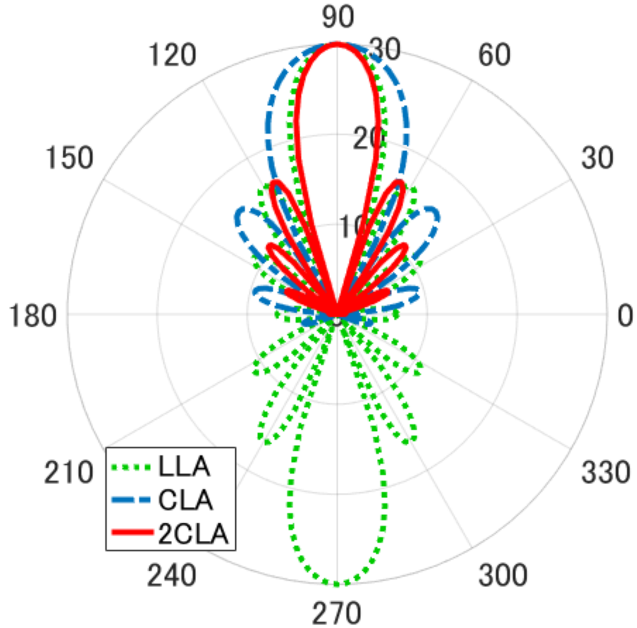
Original sound field | Reproduced by CLA | Reproduced by 2CLA

Previous Works

- DC using 2CLA w/ comparison with CLA and Linear Loudspeaker Array (LLA) (Ren & Haneda, 2019)

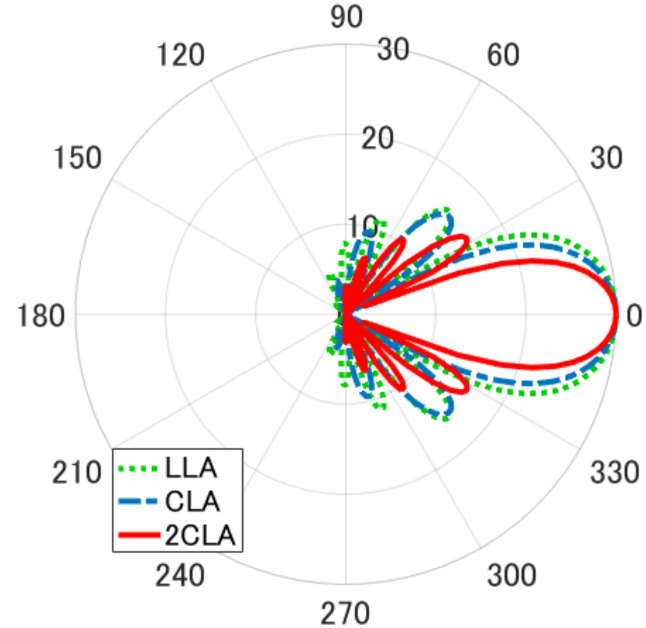
Broadside ($\varphi = 90°$)

1000 Hz



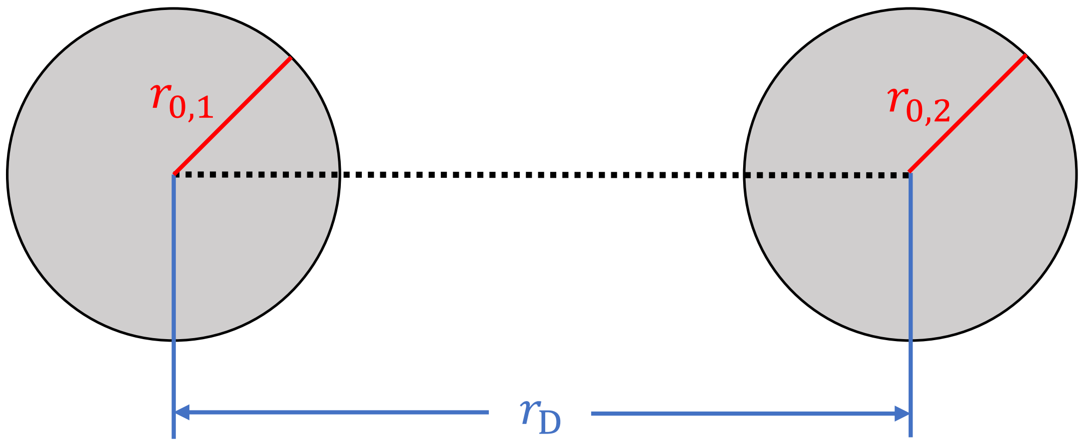
Endfire ($\varphi = 0°$)

1000 Hz



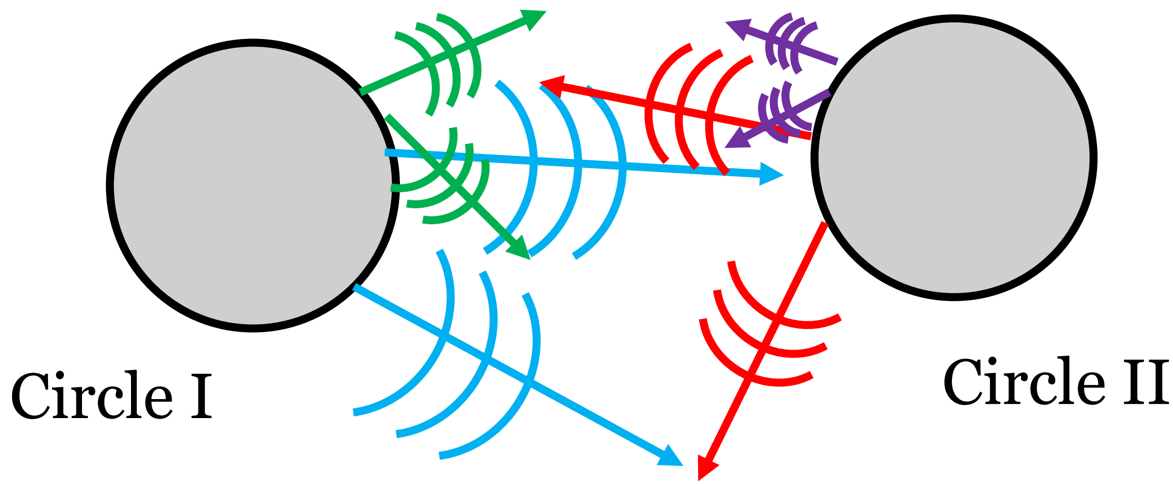
Properties

- Properties of a CLA
  - Radius r_0
  - Absorption coefficient
  - Sampling the circle
- Properties of a 2CLA
  - Radius of circle 1 $r_{0,1}$ and circle 2 $r_{0,2}$ → $r_0 = r_{0,1} = r_{0,2}$
  - Distance between the centers of the two circles r_D



Transfer function of the 2CLA

Multiple Scattering



$$G_{(\zeta)}(\boldsymbol{r}|\boldsymbol{r}') = \left(\boldsymbol{\psi}_{(\zeta)}^{\mathrm{T}} + \boldsymbol{\psi}_{(\bar{\zeta})}^{\mathrm{T}}\mathbf{T}_{(\bar{\zeta})} + \boldsymbol{\psi}_{(\zeta)}^{\mathrm{T}}\mathbf{T}_{(\zeta)}\mathbf{T}_{(\bar{\zeta})} + \cdots\right)\boldsymbol{\gamma}_{(\zeta)} \quad (1)$$

direct sound

1st reflection

2nd reflection

Transfer function of the 2CLA

$$G_{(\zeta)}(\boldsymbol{r}|\boldsymbol{r}') = \left(\boldsymbol{\psi}_{(\zeta)}^{\mathrm{T}} + \boldsymbol{\psi}_{(\bar{\zeta})}^{\mathrm{T}}\mathbf{T}_{(\bar{\zeta})} + \boldsymbol{\psi}_{(\zeta)}^{\mathrm{T}}\mathbf{T}_{(\zeta)}\mathbf{T}_{(\bar{\zeta})} + \cdots\right)\boldsymbol{\gamma}_{(\zeta)} \quad (1)$$

$\boldsymbol{\gamma}_{(\zeta)}, \boldsymbol{\psi}_{(\zeta)}$ is $(2N+1)\times 1$ vectors of $\gamma_{\nu,(\zeta)}$ and $\psi_{\nu,(\zeta)}$

$$\gamma_{\nu,(\zeta)} = -\frac{e^{-j\nu\phi'_{(\zeta)}}}{2\pi k r_{0,(\zeta)} H_\nu^{(2)\prime}\left(k r_{0,(\zeta)}\right)}, \quad \psi_{\nu,(\zeta)} = H_\nu^{(2)}\left(k r_{(\zeta)}\right) e^{j\nu\phi_{(\zeta)}},$$

$\mathbf{T}_{(\zeta)}$ is a $(2N+1)\times(2N+1)$ matrix for multiple scattering

$$T_{\nu,\mu,(1)} = -\frac{J'_\mu\left(k r_{0,(1)}\right)}{H_\mu^{(2)\prime}\left(k r_{0,(1)}\right)} H_{\mu-\nu}(k r_D)$$

$$T_{\nu,\mu,(2)} = -\frac{J'_\mu\left(k r_{0,(2)}\right)}{H_\mu^{(2)\prime}\left(k r_{0,(2)}\right)} H_{\nu-\mu}(k r_D)$$

k: wavenumber, $J_\nu(z)$: Bessel function, $H_\nu^{(2)}(z)$: Hankel function of the 2nd kind

How the r_0 and r_D matter?

I. Sound field reproduction

II. Directivity control

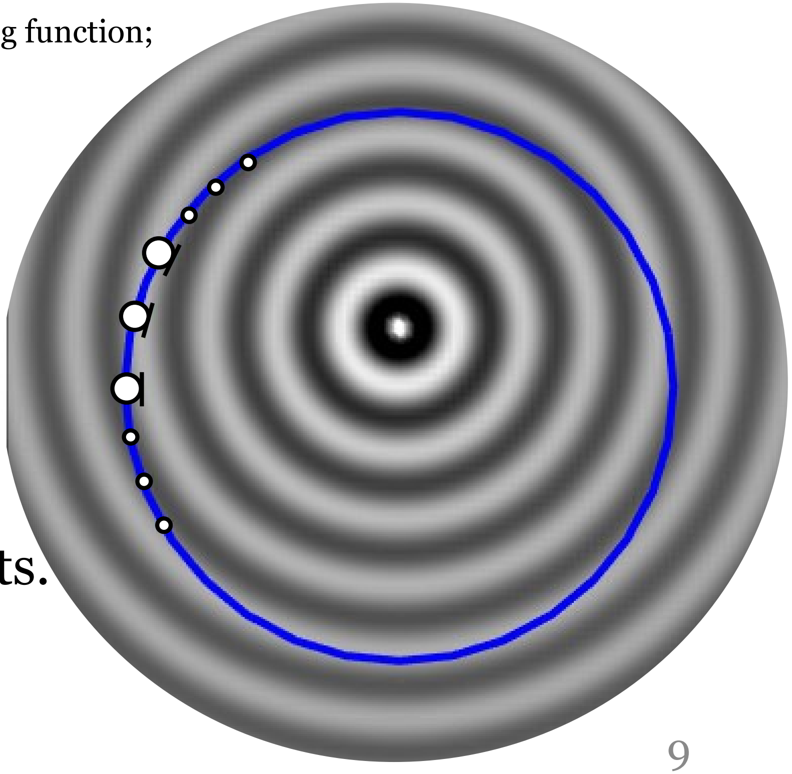
I. Sound Field Reproduction

- Pressure-matching method (PMM)

$$\mathbf{d} = \frac{\mathbf{G}^{\mathrm{H}}\mathbf{P}}{\mathbf{G}^{\mathrm{H}}\mathbf{G} + \lambda\mathbf{I}}$$

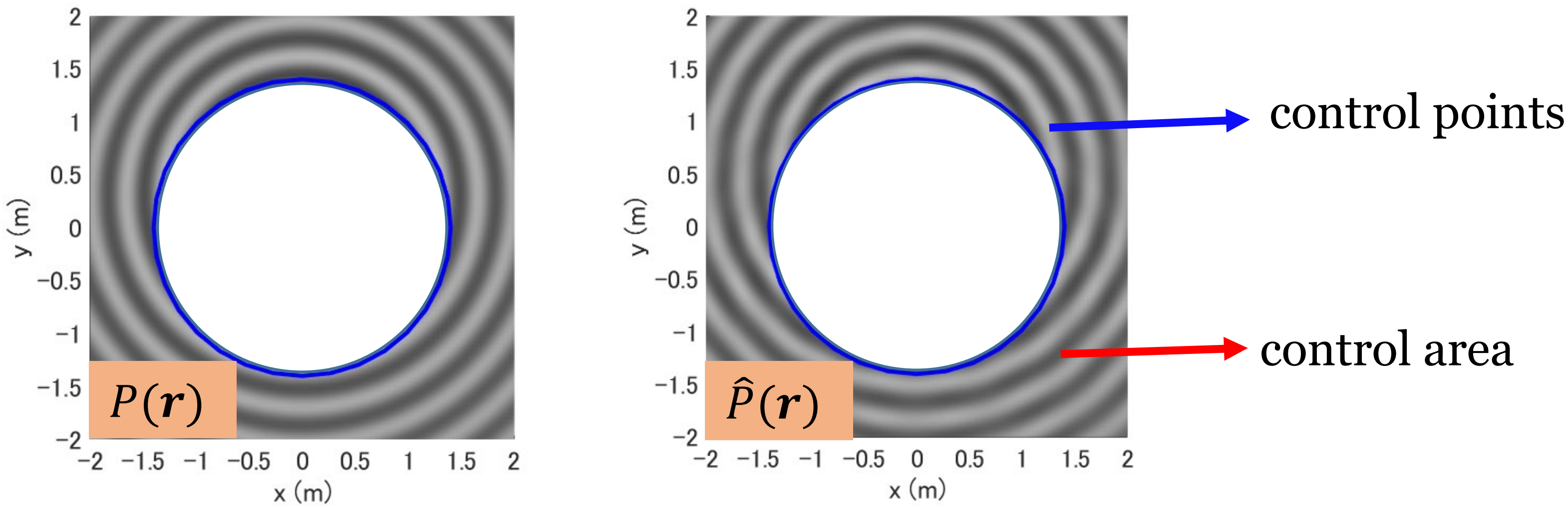
P: sound pressure at control points; **G**: transfer function; **d**: driving function;
λ: regularization parameter.

- Original sound field:
  monopole at $(0, 0.25\ \mathrm{m})$.
- Frequency domain design.
- Control area: outside the control points.



I. Sound Field Reproduction

- Evaluation



Original sound field

Reproduced sound field

- Signal-to-Distortion Ratio (SDR)

$$SDR = 10 \log_{10} \frac{\int |P(\boldsymbol{r})|^2 d\boldsymbol{r}}{\int \left|P(\boldsymbol{r}) - \hat{P}(\boldsymbol{r})\right|^2 d\boldsymbol{r}}$$

Simulation conditions

Fixed distance r_D:

$$r_0 = \begin{cases} 0.075\ \text{m} \\ 0.15\ \text{m} \\ 0.3\ \text{m} \end{cases},\ r_D = 1\ \text{m}$$

Fixed radius r_0:

$$r_0 = 0.15\ \text{m}\ ,\ r_D = \begin{cases} 0.5\ \text{m} \\ 1\ \text{m} \\ 1.5\ \text{m} \end{cases}$$

Frequencies:

$$200 - 3000\ \text{Hz}$$

Regularization parameter λ:

$$\lambda = 0 : \text{Theoretical}$$

$$\lambda = \{\lambda | W_0 = 0\ \text{dB}\} : \text{Pratical}$$

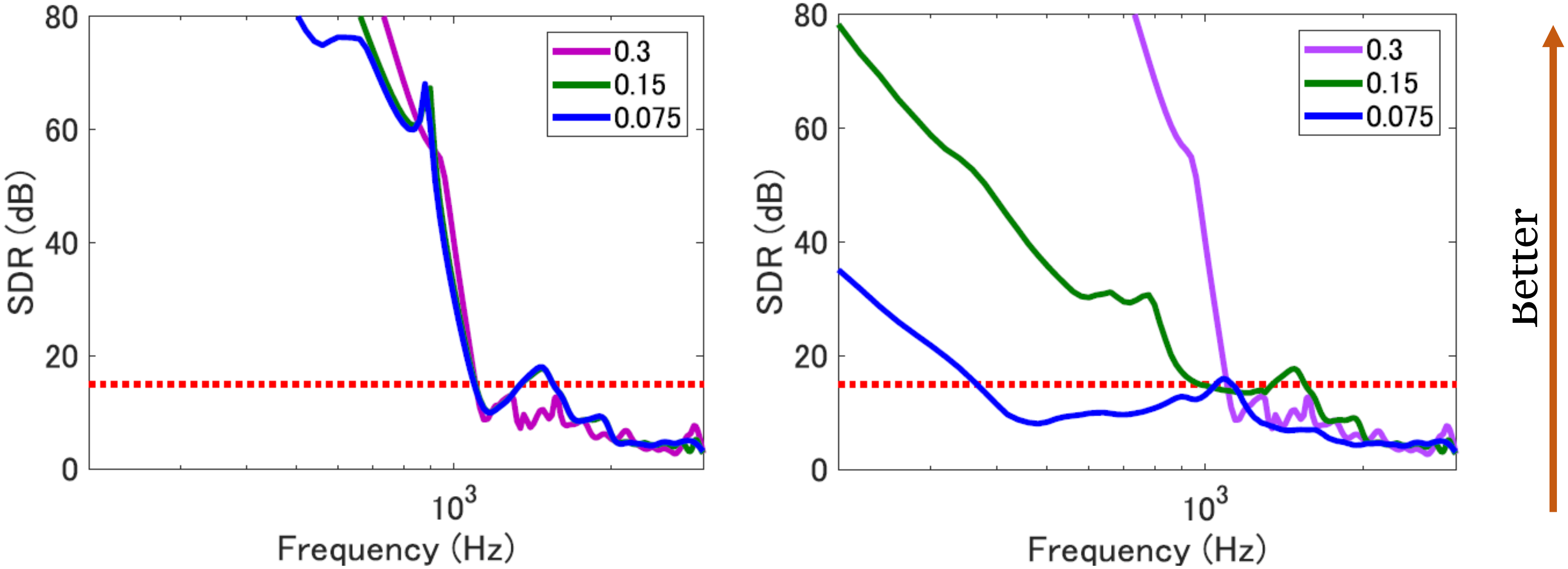
$$\text{Filter Gain } W_0 = 10 \log_{10} \|\mathbf{w}\|_2^2$$

I. Sound Field Reproduction

r_0: Doesn't matter for $\lambda = 0$.

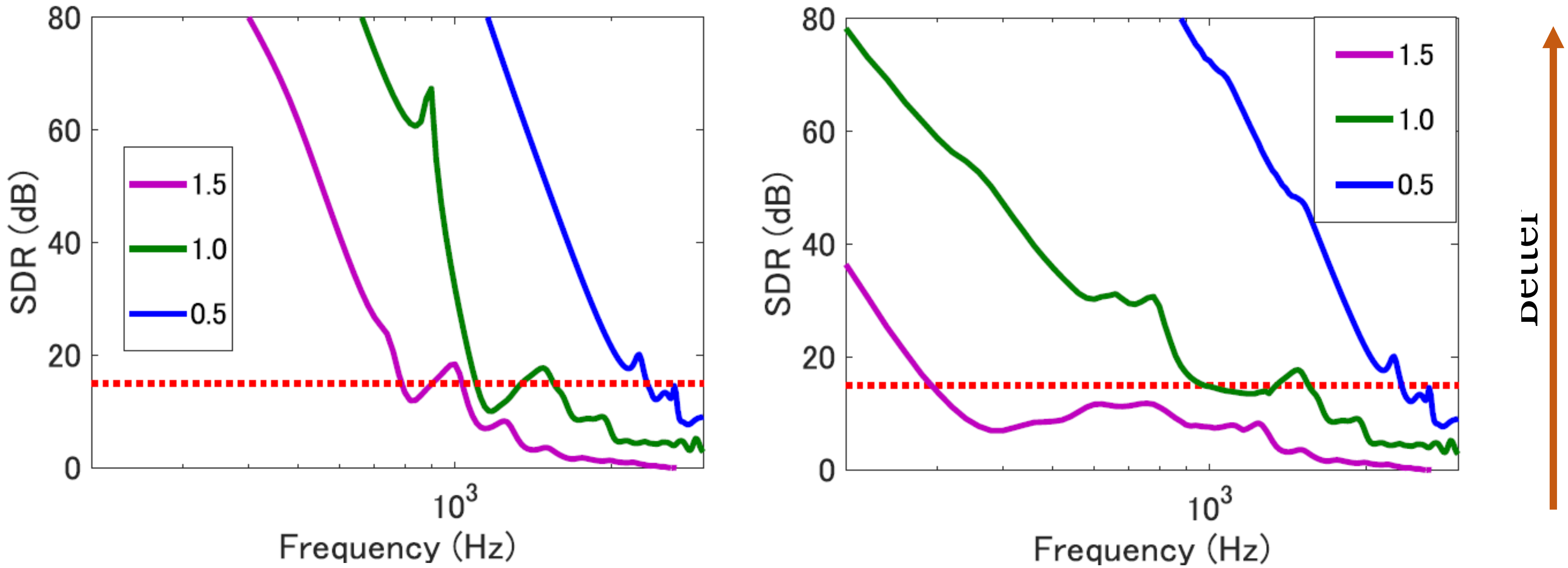
Do affect the filter gain.

Better performance with larger r_0 when filter gain is restricted.



I. Sound Field Reproduction

r_{D}: Better performance with smaller r_{D}.



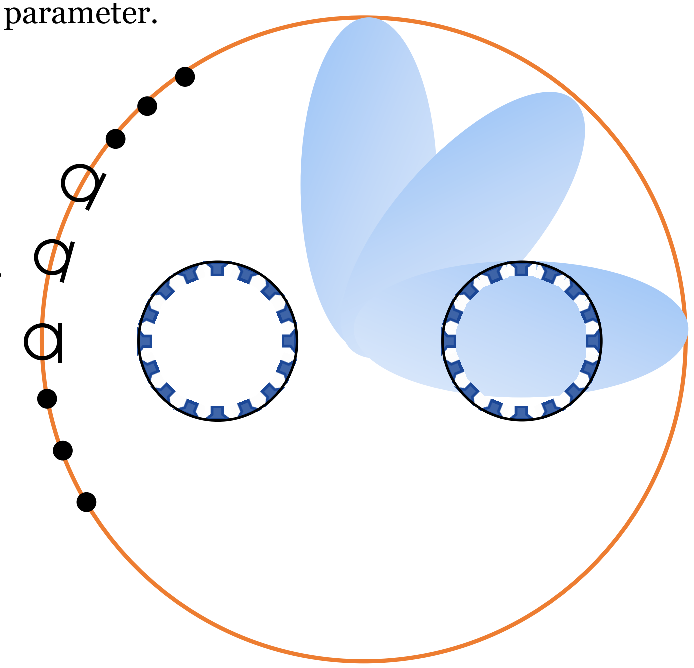
II. Directivity Control

- Minimum Variance Distortionless Response (MVDR)

$$\mathbf{w} = \frac{(\mathbf{R}^{-1} + \lambda\mathbf{I})\mathbf{C}^{\mathrm{H}}}{\mathbf{C}(\mathbf{R}^{-1} + \lambda\mathbf{I})\mathbf{C}^{\mathrm{H}}}\mathbf{f}$$

$\mathbf{w}$:Beamforming filter; $\mathbf{f}$: sound pressure at constraint points; $\mathbf{C}$: transfer function for constraint points; $\mathbf{G}$:transfer function for suppress points; $\mathbf{R} \coloneqq \mathbf{G}^{\mathrm{H}}\mathbf{G}$; λ: regularization parameter.

- Constraint point: one at look direction.
- Frequency domain design; $\mathbf{f} = \mathbf{1}$.
- Constrained filter gain $W_0 = 0$ dB.
- Suppress point: other directions.

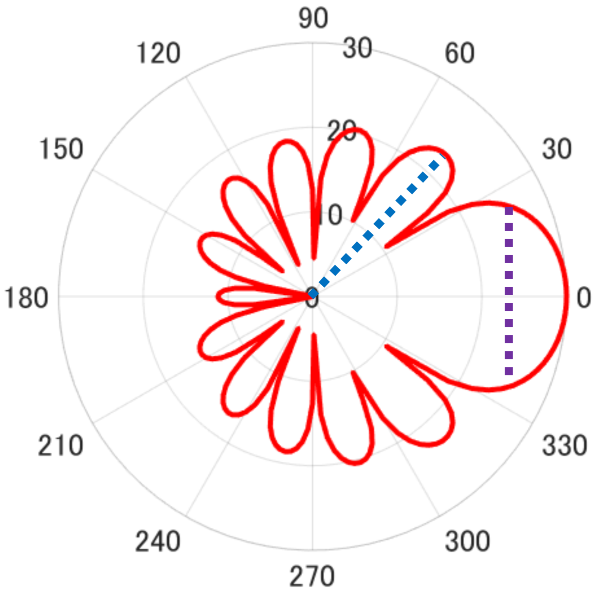


II. Directivity Control

- Evaluation
  - DI(directivity index):

$$DI = 10 \log_{10} \frac{2\pi \left\| P_{\varphi} \right\|^2}{\int_0^{2\pi} \left\| P_{\phi} \right\|^2 d\phi}$$

    - Power of the look direction
  - BW(beam width):
    - Half power beam width of the main lobe
    - The narrowness of the main beam
  - SLL(side lobe level):
    - The maximum level of the side lobe
    - Relative to the main lobe



Simulation conditions

Fixed distance r_D:

$$r_0 = \begin{cases} 0.075 \text{ m} \\ 0.15 \text{ m} \\ 0.3 \text{ m} \end{cases}, r_D = 1 \text{ m}$$

Fixed radius r_0:

$$r_0 = 0.15 \text{ m}, r_D = \begin{cases} 0.5 \text{ m} \\ 1 \text{ m} \\ 1.5 \text{ m} \end{cases}$$

Look direction:

On major axis: $\varphi = 0$; on minor axis: $\varphi = \pi/2$

Frequencies:

$$300 \text{ Hz}, 1000 \text{ Hz}$$

II. Directivity Control

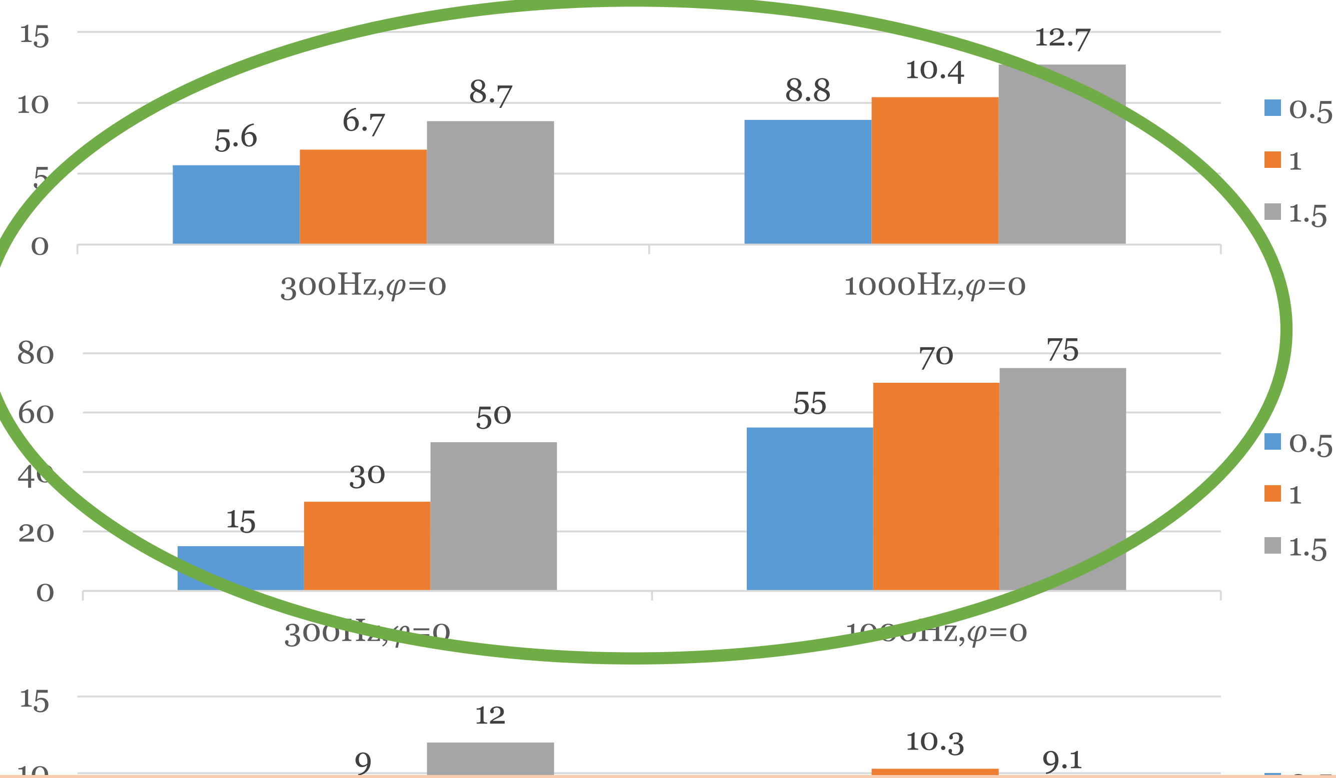
- Fixed distance r_D

$r_0 \in \{0.075\ \mathrm{m}, 0.15\ \mathrm{m}, 0.3\ \mathrm{m}\}$, $r_\mathrm{D} = 1\ \mathrm{m}$

Major axis

DI

$90° - BW$



Better

r_0: Better performance with greater r_0.

II. Directivity Control

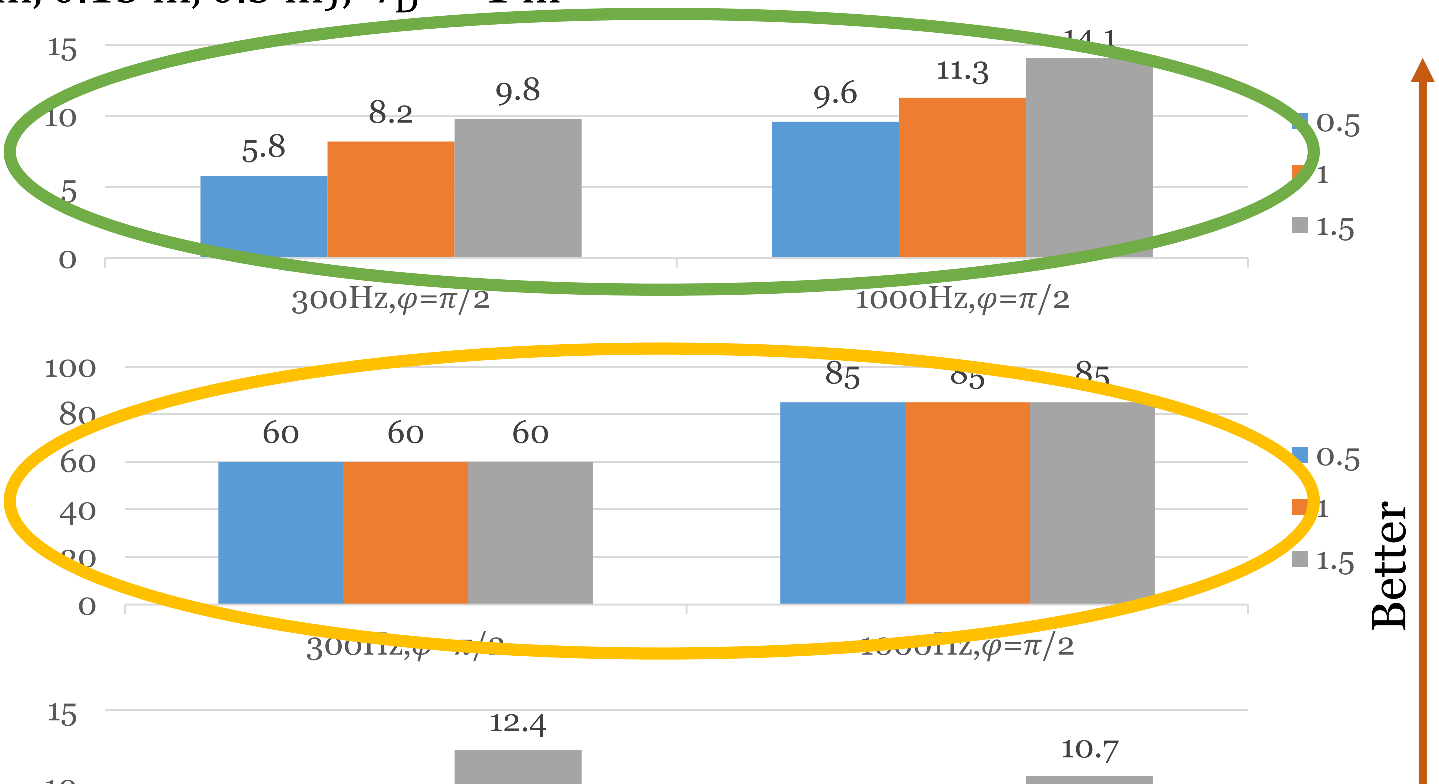
- Fixed distance r_D

Minor axis

$r_0 \in \{0.075\ \mathrm{m}, 0.15\ \mathrm{m}, 0.3\ \mathrm{m}\}$, $r_\mathrm{D} = 1\ \mathrm{m}$

DI

$90° - BW$



r_0: Better performance with greater r_0.
Doesn’t affect beam width.

II. Directivity Control

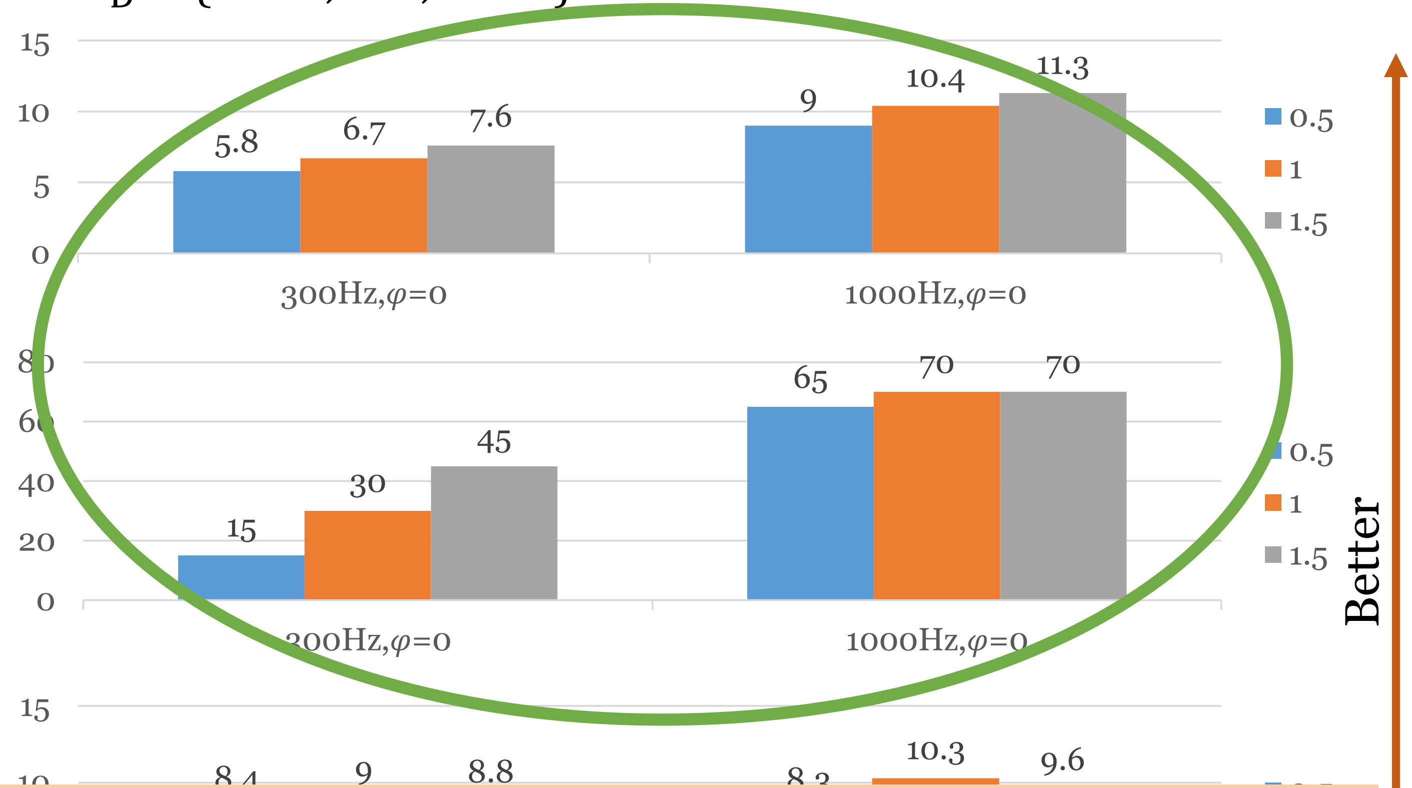
- Fixed radius r_0

$r_0 = 0.15$ m, $r_D \in \{0.5 \text{ m}, 1 \text{ m}, 1.5 \text{ m}\}$

Major axis

DI

$90° - BW$



r_D: Better performance with greater r_D.

II. Directivity Control

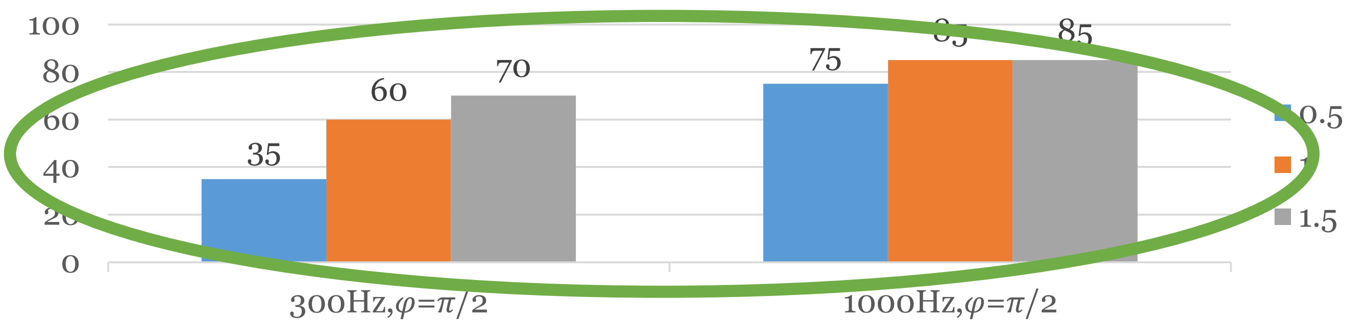
- Fixed radius r_0

Minor axis

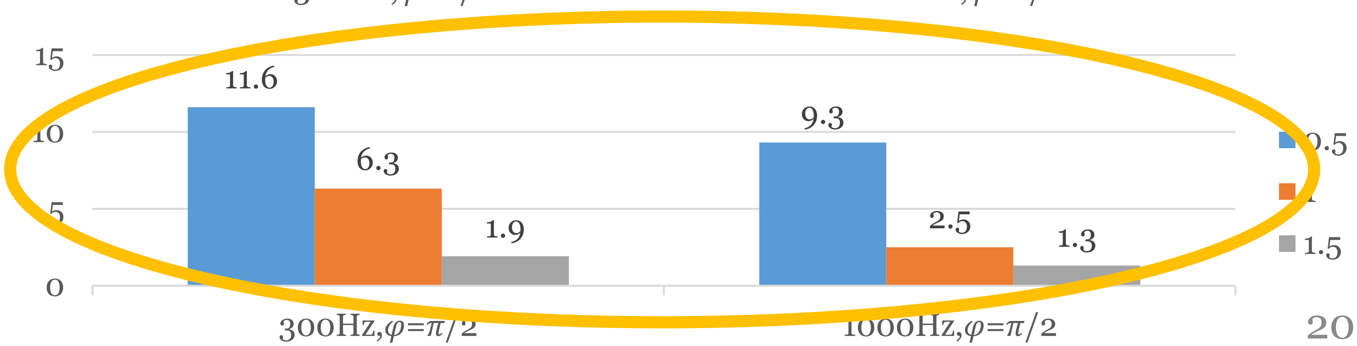
$r_0 = 0.15$ m, $r_\mathrm{D} \in \{0.5\text{ m}, 1\text{ m}, 1.5\text{ m}\}$

r_D: Better BW but worse SLL with greater r_D.
Greater r_D may lead to aliasings.

$90° - BW$



$-SLL$



Better

Conclusion

- r_0 and r_D do affect the performance of the 2CLA
  - Greater r_0 is supposed to get better performance when filter gain constrained
  - Greater r_D is better for controlling the sound on the major axis side while may lead to aliasing for the minor axis side

- Further works
  - Different $r_{0,(1)}$ and $r_{0,(2)}$, other methods, etc.
  - Reasons